working paper
department
of economics

AGGREGATE PRODUCTION FUNCTIONS AND THE EXPLANATION OF WAGES: A SIMULATION EXPERIMENT*

by

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No. 61 October, 1970

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* This research was supported by NSF Grant #GS-1791. I am indebted to Nancy Greene and, especially, Hossein Askari for programming and computational assistance.

AGGREGATE PRODUCTION FUNCTIONS AND THE EXPLANATION OF WAGES: A SIMULATION EXPERIMENT

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1. Introduction

Recent work[1] has shown pretty clearly that the conditions under which the production possibilities of a technologically diverse economy can be represented by an aggregate production function are far too stringent to be believable. This is true not only of the conditions for the existence of an aggregate capital stock, where what is required is that the production functions of individual firms differ at most by capital-augmenting technical differences, but also of the existence of labor and output aggregates, where every firm must hire the same proportions of each type of labor and produce the same market basket of outputs. Moreover, the view that aggregate production functions are only approximations anyway cannot be sustained merely because such approximations are required only to hold over a limited range of the variables.[2]

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[1] Summarized in Fisher [7]. Relevant pieces include Diamond [2]; Fisher [3], [4], [5], and [6]; Gorman [8]; Solow [10] and [12]; and Whitaker [13] and [14], among others.

[2] See Fisher [6].

Yet aggregate production functions apparently work nevertheless and do so in a way which is prima facie not easy to explain. It is easy enough to understand why, in economies in which things move more or less together, a relationship giving an aggregate measure of output as dependent on aggregate measures of capital and labor should give a good fit when applied to the data. What is not so easy to explain is the fact that the marginal product of labor in such an estimated relationship appears to give a reasonably good explanation of wages as well. In its simplest form, this puzzle is set by a remark which Solow once made to me that had Douglas found labor's share to be 25% and capital's 75% instead of the other way round, we would not now be discussing aggregate production functions.

If the fact that estimated aggregate production functions explain wages fairly well is a statistical artifact, then it is certainly not an obvious one. There seems little reason why a function fitted to time series of output and input data on an aggregate basis should have this property; yet coincidence seems hard to swallow.[3]

This paper reports on a simulation experiment which may cast some light on this issue. Aggregate Cobb-Douglas production functions were

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[3] Phelps Brown [9] simply dismisses the time series results as poor or implausible, largely because of their failure to allow for technical change. In the light of Solow's seminal paper [11] and its successors, this can no longer be done. Phelps Brown's arguments as to cross-section estimates explain nothing about the time series results, nor do they show why a cross-sectionally estimated production function should give reasonable wage predictions for years far from that of the original cross section.

estimated for data produced by numerous fictitious and very simple economies in which there was only one homogeneous output and one homogeneous kind of labor but in which the conditions for the existence of an aggregate capital stock were definitely violated. Various measures of the performance of such a function were computed, among them measures of how well the aggregate production function did in explaining the history of wages. The histories of the economies were varied in a number of ways and the performance measures compared to see under what circumstances the aggregate production function did well.

Naturally, the results of such experiments can only be suggestive. Moreover, since the entire economy is under the control of the investigator, any organizing principle which appears in the results and which can be heuristically argued to be reasonable can also be argued, with the benefit of hindsight to have been fairly obvious.

Nevertheless, the principal result of this investigation, while only suggestive, is very suggestive indeed. It is obvious that, since a Cobb-Douglas production function implies the constancy of labor's share, such a function cannot be expected to explain wages well in an economy in which that share is not constant. What is not obvious is that in economies in which labor's share _happens_ to be roughly constant, even though the true relationships are far from yielding an aggregate Cobb-Douglas, such an aggregate production function _will_ yield a good explanation of wages. Yet this is generally what we find for our fictitious economies and the relationship between the variance in labor's share and the goodness-of-fit of the wage predictions is close, although certainly not

perfect.

If such a result holds for other than our simplified and fictitious economies, it has important implications. It suggests that the view that the constancy of labor's share is due to the presence of an aggregate Cobb-Douglas production function is mistaken. Causation runs the other way and the apparent success of aggregate Cobb-Douglas production functions is due to the relative constancy of labor's share. The explanation of such constancy remains to be found. Our results suggest that should the forces making for it suddenly change, then aggregate Cobb-Douglas production functions would cease to give reasonable wage explanations.

In a word then, the answer suggested by the present results to Solow's question is that an aggregate Cobb-Douglas production function estimated from input and output data does well in wage prediction not because wages are generated from it but because the behavior of labor's share just happens to approximate the central stylized fact generated by such a function, even though the mechanism actually generating wages and output is rather different. Such a view obviously has extensions to the performance of aggregate production functions other than Cobb-Douglas.

2. The Model

Our economies each consist of n units which hire the same kind of labor and produce the same kind of output. We shall refer to these units as "firms," although they might equally well or better be thought of as industries each of which consists of a number of identical firms. In our experiments, n was taken to be 2, 4, or 8. Much higher numbers would have

led to greatly increased computing time and the results do not suggest that it would have made much difference.

Each firm has a different kind of capital stock and its technology is embodied in that stock. Thus different firms have different production functions and capital is not transferable among firms (although labor is). The ith firm's production function is given by:

$$y_i(t) = A_i(t)\, L_i(t)^{\alpha_i}\, K_i(t)^{1-\alpha_i} \qquad (i = 1, \ldots, n). \tag{2.1}$$

Here, K_i is the amount of the ith firm's capital; L_i the amount of labor employed by the firm; and y_i, the amount of output it produces. Calendar time is denoted by t (running from 1 to 20 in each economy); $A_i(t)$ is a function of time representing disembodied Hicks-neutral technical change (which, given the fact that the production functions are Cobb-Douglas, is indistinguishable from disembodied factor-augmenting change)[1]; α_i is a parameter.

Cobb-Douglas production functions for individual firms were used both because they allow great simplifications in the labor-allocation algorithm described below and because they ought to provide the best chance for an aggregate Cobb-Douglas to work well.

At any moment of time, the aggregate labor force employed is, of course,

$$L(t) \equiv \sum_{i=1}^{n} L_i(t) \quad . \tag{2.2}$$

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[1] Embodied technical change, of course, is, in a sense, what the whole model is about.

Total output produced is

$$(2.3) \qquad Y(t) \equiv \sum_{i=1}^{n} y_i(t) \quad .$$

However, whereas $L(t)$ is one of the givens of the economy, $Y(t)$ is not, since it obviously depends on the way in which employment is allocated to firms.

Such allocation is performed so as to make total production efficient. As would a perfect labor market or a socialist planning board, the $L_i(t)$ are chosen each period so as to maximize $Y(t)$, given $L(t)$, $K_1(t),\ldots, K_n(t)$, $A_1(t),\ldots, A_n(t)$, and (2.1) and (2.2). We denote output so maximized as $Y^*(t)$.

Now, it is not hard to show that if all the α_i were the same, then Y^* would be given by an aggregate Cobb-Douglas production function:

$$(2.4) \qquad Y^*(t) = A(t)\, L(t)^{\alpha}\, J(t)^{1-\alpha}$$

where $J(t)$ depends only on the $K_i(t)$ and the $A_i(t)$ and not on $L(t)$. (This would not be true, incidentally, of $Y(t)$.) The results on the existence of aggregate production functions, already referred to, imply that this will certainly not be the case if the α_i are different for different firms. Indeed, they show that there exists no aggregate production function:

$$(2.5) \qquad Y^*(t) = F(J(t), L(t), t)$$

in which $J(t)$ is merely a capital index independent of labor.[2] For each

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[2] See Fisher [3].

economy, however, we ignore this and estimate an aggregate Cobb-Douglas production function in the form (2.4) with $J(t)$ chosen as about to be described. As already stated, the relative performance of such estimated functions is compared across economies.

A single experiment, then, consists of choosing values for the parameters and of generating a twenty-year time series for $L(t)$, $K_1(t), \ldots, K_n(t)$, and $A_1(t), \ldots, A_n(t)$.[3] For each year, the $L_i(t)$ are chosen so as to maximize $Y(t)$ subject to (2.2).[4] Given the labor assignments, wages and capital rentals for the n different capital types are computed by taking marginal products in the individual production functions. We denote by $w(t)$ the resulting wage at time t and by $r_i(t)$ the marginal product of the ith capital type at that time.

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[3] Note that the individual capital stocks are given exogenously.

[4] This was done as follows, making use of the property that with Cobb-Douglas production functions every $L_i(t) > 0$. It is easy to write an algorithm which equalizes (by iteration) the marginal product of labor in any given pair of firms, given the total amount of labor employed by those firms. When $n = 2$, this is all there is to it, since equality of marginal product is obviously the condition for efficient allocation. For $n > 2$, we first divided labor equally among all n firms; then we took the total labor assigned to firms 1 and 2 and reallocated it between the firms to equalize marginal product. The next step took the labor then allocated to firms 2 and 3 and reassigned it to them efficiently; then we went on to firms 3 and 4, and so forth up to firms n-1 and n. The process was then repeated starting at firms 1 and 2 and continued until all marginal products were equalized (approximately). It is easy to prove convergence of this algorithm and, although it is not particularly efficient, it did not take a prohibitive amount of computer time for $n = 8$.

Now, it is obvious from Euler's theorem that:

$$Y^*(t) = w(t)\ L(t) + \sum_{i=1}^{n} r_i(t)\ K_i(t) \qquad (t = 1,\ldots, 20). \tag{2.6}$$

This means that at any moment of time, the sum of the right-hand side of (2.6) makes an excellent capital index. The problem, of course, occurs because the relative magnitudes of the $r_i(t)$ not only do not remain constant over time but also are not independent of the magnitude of $L(t)$; this is the essence of the capital-aggregation problem.

Nevertheless, it seems clear that an aggregate production function will do best if its capital index comes as close as possible to weighting different capital goods by their rentals. Accordingly, we constructed $J(t)$ for use in (2.4) by

$$J(t) \equiv \sum_{i=1}^{n} \bar{r}_i K_i(t) \qquad (t = 1,\ldots, 20), \tag{2.7}$$

where

$$\bar{r}_i \equiv (1/20) \sum_{t=1}^{20} r_i(t) \qquad (i = 1,\ldots, n). \tag{2.8}$$

3. The Experiments: Detailed Description

Obviously, crucial questions are the choice of the parameters and time series which define an experiment and the choice of measures of performance of the aggregate production function. In the present section, we discuss the former question, taking up the latter in the next section.

We begin with the crucial parameters of the production functions, α_i. As aggregate estimates for the United States tend to show a Cobb-Douglas

exponent for labor of about .75, we chose the α_i around this value. In one set of experiments, the α_i were chosen in the range .7 -- .8; in a second, they were chosen in the range .6 -- .9. In both cases, the n individual α_i were chosen so that α_1 was at the lower and α_n at the upper endpoint of the range, with the remainder (if any) of the exponents uniformly spread over the range. Thus, for example, in the .6 -- .9 case, with n = 4, the exponents were .6, .7, .8, .9. Note that in all cases, the unweighted average of the α_i was kept constant at .75.

As already indicated, n was chosen at 2, 4, or 8. A little reflection, however, shows that changing the number of firms represented in this model is not merely a matter of changing n, despite the terminology. One has two choices in increasing the number of firms. The first of these is to add firms without altering the set of α_i to be examined. Given constant returns and efficient labor allocation, however, adding a firm with a labor exponent identical to that of an existing firm is equivalent to increasing the size of the existing one by giving it more capital stock. Since one of the other choices described below involves doing just this, changes in n are not the only way in which the experiments reflect changes in the number of firms.

Aside from this, however, a change in n which is not merely equivalent to an increase in firm size must involve a change in the set of α_i. One can not merely change n alone. The way in which increases in n were reflected in the set of α_i has already been indicated. Note that whereas the average α_i is always the same, an increase in n involves a reduction in the variance of the α_i around their mean, other things equal. (Weight-

ing by firm size, of course, that variance can increase when we increase the number of firms by changing firm size at given α_i; so can the weighted mean.) An increase in n involves more differnet firms, but also involves less polarization among firms.

The three sets of exogenously specified time profiles were all chosen to be approximately exponential trends. Thus:

(3.1) $$\log L(t) = \lambda_0 + \lambda_1 t + \lambda_2 \varepsilon_t \quad ;$$

(3.2) $$\log K_i(t) = \beta_{i0} + \beta_{i1} t + \beta_{i2} \eta_{it} \qquad (i = 1,\ldots, n);$$

(3.3) $$\log A_i(t) = \gamma_{i0} + \gamma_{i1} t + \gamma_{i2} \nu_{it} \qquad (i = 1,\ldots, n),$$

where the λ_j, β_{ij}, and γ_{ij} are parameters and ε_t, the η_{it}, and the ν_{it} are independently distributed standard normal deviates. The random elements (which were kept small) were introduced partly for minor reasons of realism, partly to see what difference they made, and partly to avoid multicollinearity in the estimation of the aggregate production function. This is particularly important in the cases in which allowance for exponential disembodied technical change is made in estimating (2.4), since the variables in that regression will be log (J(t)/L(t)) and t. Moreover, as reported below, some regressions were made without imposing constant returns, so that the variables in such cases would be log J(t), log L(t), and t.

After considerable gross experimentation to discover what parameter choices made much difference, the random terms were standardized for the main experiments reported here by always taking $\lambda_2 = .02$, $\beta_{i2} = .0001$, and $\gamma_{i2} = 0$ $(i = 1,\ldots, n)$.

Further, λ_1 was chosen as .03 and λ_0 as zero. Thus, labor always grows at an average 3% trend with deviations normally distributed with mean zero and standard deviation .02. The initial experimentation referred to showed the measures of performance of the aggregate production function not to be terribly sensitive to choices of λ_0 and λ_1, so these were standardized to reduce the gigantic number of different cases examined. For similar reasons, all the γ_{i0} were chosen as 0. (This is merely a choice of units.)

The performance measures are sensitive, as one might expect, to the choices made as to the capital and technical change parameters, the principal sensitivity being to the trend terms. If all firms are growing at the same rate, then an aggregate capital index can be expected to perform rather well; if firms grow at rather different rates, this is less likely to be the case.

The initial conditions for the K_i were chosen as follows. The n firms were divided into two halves, one with relatively low values of α_i and one with relatively high values. We shall refer to these as "low-α" amd "high-α" firms, respectively. The β_{i0} were chosen in three sets. In the first of these, all $\beta_{i0} = 0$ $(i = 1, \ldots, n)$. In the second set, all β_{i0} for the low-α firms were set at 0 and the β_{i0} for the high-α firms were set at 2. In the third set, these values were reversed. As already remarked, these choices can also be thought of as changing the number of firms. Since all logarithms were natural, changing a β_{i0} from 0 to 2 amounts to multiplying the initial capital stock by something over 7.

The selection of the trend parameters was done rather more finely. In one set of experiments, which I shall refer to as "two-group capital" experiments, β_{i1} was set at 0 for firms in the high-α group, while all β_{i1} for firms in the low-α group were set equal to each other and, in successive runs allowed to vary from -.05 to +.05 in steps of .01.[1]

Similarly, in a second set of experiments, which I shall call "two-group Hicks" experiments, the γ_{i1} for the high-α firms were set at 0. To secure comparability with the two-group capital experiments, however, the γ_{i1} for the remaining firms were chosen so that the equivalent trends in their capital stocks would all be equal and move in steps of .01 from -.05 to +.05. Such equivalent trends, of course, are such that a choice of a particular value for β_{i1} corresponds to a choice of $\gamma_{i1} = \beta_{i1}(1 - \alpha_i)$. We also report results for preliminary runs of the two-group Hicks experiments, in which the γ_{i1} for the low-α firms were chosen all equal and moving from -.05 to +.05 in steps of .01.

Now, since in these two-group experiments, all firms in a given group grow or shrink together, there is a real sense in which they are roughly all one firm for our purposes, so that, were these the only experiments performed, we would not be giving too much scope to the effects of increases in n. This is particularly so, since the way in which the α_i were chosen, as already described, means that the average α_i for each of the two groups in such an experiment is getting closer to the other one when n increases.

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[1] Note that, in view of constant returns, there is no essential difference between allowing one group of firms to shrink and allowing the other to grow.

Accordingly, we performed two more sets of experiments, which I shall refer to as "fanning capital" and "fanning Hicks" experiments, respectively.

In the fanning capital experiments, β_{i1} was set equal to zero for the firm with the highest α_i. Call this firm 1 and number the firms so that α_i decreases.[2] The remaining β_{i1} were chosen as:

(3.4) $$\beta_{i1} = (i - 1)\mu \qquad (i = 1,\ldots, n)$$

where μ was allowed to go from -.05 to +.05 in steps of .01. Thus the rates of growth of the firms "fanned out" from that of the first firm. Note that much bigger relative rates of growth are involved here (for $n > 2$) than in the case of the two-group experiments.

The fanning Hicks experiments were chosen analogously. In these,

(3.5) $$\gamma_{i1} = (1 - \alpha_i)(i - 1)\mu \qquad (i = 1,\ldots, n)$$

with μ chosen as before.

Since the number of separate experiments to be run would be gigantic if all possible combinations were tried, all γ_{i1} were set at zero in performing capital experiments and all β_{i1} were set at zero when performing Hicks experiments.

The number of different experiments was still high. Each of the two-group sets (including the preliminary two-group Hicks set) and each of the

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[2] Thus the high-α firms are firms $1,\ldots, n/2$ and the low-α ones are firms $n/2+1,\ldots, n$.

fanning sets involves 11 experiments for each choice of the other parameters. Since there are 3 choices for n, 2 choices of the range of the α_i, and 3 choices of initial conditions for the capital stocks, each five sets of experiments involves 198 separate runs. In addition, to ensure that the random variation involved had little effect on the conclusions, we performed 5 runs instead of 1 each time all the β_{i1} were set at zero in the capital experiments or all the γ_{i1} were set at zero in the Hicks experiments. This brought the total number of runs to 202 for each of the five sets or a grand total of 1010 exclusive of the preliminary experimentation referred to earlier.[2] However, since for n = 2 a fanning experiment is identical with a two-group experiment, the actual total of runs, exclusive of the extensive preliminary experimentation, was 830.

The various choices of parameters are summarized in Table 3.1.

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[2] Since, as it turned out, there was only negligible difference in the results of the experiments with zero growth which differ only by random variation, we used only one of the five sets each time in analyzing the results as reported below.

Table 3.1

Different Choices of Parameters for Experiments

Parameters	Choice of Values
n	2; 4; 8.
α_i	evenly distributed over range .7 - .8 or .6 - .9.
λ_0	0.
λ_1	.03.
λ_2	.02.
β_{i0}	0 all firms; 0 low-α firms and 2 high-α firms; 0 high-α firms and 2 low-α firms.
β_{i1}	0 all firms in Hicks experiments; in two-group capital experiments, 0 for high-α firms and μ for low-α firms; in fanning capital experiments, $(i - 1)\mu$.
β_{i2}	.0001 all firms.
γ_{i0}	0 all firms.
γ_{i1}	0 all firms in capital experiments; in final two-group Hicks experiments, 0 for high-α firms and $(1 - \alpha_i)\mu$ for low-α firms; in preliminary two-group Hicks experiments, 0 for high-α firms and μ for low-α firms; in fanning Hicks experiments, $(1 - \alpha_i)(i - 1)\mu$.
γ_{i2}	0 all firms.
μ	-.05, -.04,..., 0,..., .04, .05.

4. Aggregate Regressions and Measures of Performance

In each of the experiments, four regressions were performed on the aggregate data. In two of these, constant returns was imposed and in two no such imposition was made; further, given the imposition or non-imposition of constant returns, regressions were run both with and without a trend term allowing for Hicks-neutral disembodied technical change. This was done both for Hicks experiments in which there was such change at the firm level and for capital experiments in which there was not.

Thus the four equations estimated, all versions of (2.4), were (omitting the time argument):

$$\log Y^* = a + b \log J + c \log L \quad ; \tag{4.1a}$$

$$\log Y^* = a + b \log J + c \log L + d\ t \quad ; \tag{4.1b}$$

$$\log (Y^*/L) = a + b \log (J/L) \quad ; \tag{4.1c}$$

and

$$\log (Y^*/L) = a + b \log (J/L) + d\ t \quad . \tag{4.1d}$$

We now turn to the question of how the performance of such estimated aggregate production functions ought to be measured.

The first such measure is an obvious one; it is the R^2 of the regressions. While we should not expect such correlations to be low so long as aggregate capital, J, is correlated with its components, it remains a necessary feature of the good performance of an aggregate production function that when fitted to aggregate output and input data it yield a close fit

in explaining the dependent variable.[1]

A second natural measure is relevant to the equations in which constant returns is not imposed. The underlying structure certainly is constant returns. It is therefore of some interest to know whether or not an aggregate production function when estimated from data generated by that structure reveals that fact.

Somewhat similarly, in the Hicks experiments, there is underlying Hicks-neutral technical change and in the capital experiments there is not. It is of some interest to see whether this will be detected in the aggregate estimates.

Further, the underlying firm production functions are all in fact Cobb-Douglas with or without Hicks-neutral technical change. In general, we would expect a well-performing aggregate Cobb-Douglas production function to give parameter estimates which are averages of the individual firm production function parameters.

All of this, however, while interesting, is subsidiary to our main focus, that of the explanation of wages. Our principal performance measure, therefore, will measure the degree of success of the aggregate production functions in explaining the generated wage data, to which they are not directly fitted. What measure should be used?

The measure which first naturally suggests itself is the squared coefficient of determination:

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[1] R^2 is a sensible measure of goodness of fit here. t-statistics are not, since the problem is non-stochastic.

$$I^2 = 1 - \frac{\sum_t (w(t) - \hat{w}(t))^2}{\sum_t (w(t) - \bar{w})^2} \quad , \tag{4.2}$$

where $w(t)$ is the actual wage, $\hat{w}(t)$ the predicted wage,[2] and $\bar{w}$ the average actual wage over the twenty years. This has the usual property of having an upper bound of unity, although it is not bounded below by zero (or anything else).

A little thought (and some hindsight), however, suggests that this is not a terribly appropriate measure for our purposes. There is no reason why our experiments cannot generate economies in which the variance of wages is very small. This would make I^2 also very small, even if predicted wages came very close to actual ones. This possibility occurs in practice in a way which is convincing as to the inappropriateness of I^2. There are numerous cases in which, when performing a given series of experiments which differ only in relative growth rates (μ) but have identical initial conditions and other parameters, I^2 suddenly becomes very

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[2] There is an apparent issue as to how $\hat{w}(t)$ should be computed. From (2.4),

$$\hat{w}(t) = \alpha(Y^*(t)/L(t)) \quad , \tag{4.3}$$

where α, of course, is estimated from the aggregate regressions (4.1). Should $Y^*(t)$ in (4.3) be taken as the actual value of $Y^*(t)$ or the predicted value? Fortunately, R^2 for all the regressions is so very high that it turns out to make no practical difference which one is used. In the results reported below, we used the actual value of $Y^*(t)$.

negative at a particular value of μ while being satisfactorily positive on both sides of that value. Examination of the histories of the correspon-ding economies reveals that for smaller values of μ, say, wages are going up and, for larger values, they are going down, while, for the critical value of μ, they are nearly constant. Predicted wages pretty much dupli-cate that behavior, and the numerator of the fraction in (4.2) stays small, so that the erratic behavior of I^2 is entirely due to the behav-ior of the denominator and the accident that wages happen to have a small variance in a particular experiment.

This naturally suggests the use of the numerator in (4.2) as itself the measure of performance as to wage explanation, and the natural form in which to use this is as the root-mean-square prediction error. Since this is not scale free, we used instead the relative root-mean-square error, calculated as:

$$(4.4) \qquad S = \frac{\sqrt{(1/20) \sum_t (w(t) - \hat{w}(t))^2}}{\bar{w}}$$

as our primary measure of performance.[3]

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[3] There are other associated measures which might be used. One might, for example, regress $w(t)$ on $\hat{w}(t)$ and observe whether the constant term was close to zero and the slope close to unity as well as computing the correlation coefficient. While we did in fact do this, the results did not seem to provide much information not already contained in the relative root-mean-square error and they are not reported here. In any case, the relative root-mean-square error seems the most natural single measure.

5. Subsidiary Results

As more or less suspected, R^2 turns out not to be a particularly sensitive measure of performance. Essentially without exception, it is very high, generally around .99. This holds for all four of the regressions (4.1) and all sets of experiments. It reflects the fact that with everything moving in trends of one sort or another, an excellent fit is obtained regardless of misspecification of different sorts.

The estimated coefficients, however, do reflect misspecification in different ways. Without going into great detail, the following comments convey the general impression given.

Consider first cases in which the correct specification as to the presence or absence of Hicks-neutral technical change is made. In such cases, it often makes a great difference whether or not constant returns is imposed, particularly in experiments where things are moving about. Not only do the non-constant returns regressions (4.1a) and (4.1b) often yield estimates of the degree of homogeneity $(b + c)$ which are widely different from unity, but also there are numerous (but less frequent) cases in which the individual coefficients are ridiculous, including many cases in which marginal products are estimated as negative. On the other hand, there are many cases in which the individual coefficients are plausible and some in which they add up to something around unity.

If constant returns is imposed, however, such wild behavior essentially completely disappears and the exponents of labor and capital in the aggregate regressions almost always lie in the range covered by the

corresponding firm coefficients.

One obvious explanation for such behavior is that of multicollinearity. $J(t)$ is composed of several variables trending at different rates with small random fluctuations; $L(t)$ is such a trend variable with larger fluctuations. It is not then too surprising that when both $\log J(t)$ and $\log L(t)$ appear in the regressions, particularly when t itself does, results as to individual coefficients look peculiar. When only $\log (J(t)/L(t))$ appears, one would expect to do better in this respect. Still, there is in many cases nothing like exact collinearity so this may only be part of the explanation.

Multicollinearity may also play some part in the explanation of a second phenomenon, this one relating to the correct or incorrect specification of the presence of Hicks-neutral technical change. Concentrating on the constant returns results, we find, as one might expect, that when dealing with Hicks experiments it makes a considerable difference whether or not we allow for technical change at the aggregate level. If we do, then results look plausible and, for low levels of movement, the relative root-mean-square error in the prediction of wages (S) is low. If no allowance for technical change is made in such cases, results are far worse.

What is somewhat more surprising than this, however, is the fact that in the capital experiments when there is no underlying technical change, results are nevertheless somewhat better when technical change at the aggregate level is assumed to be absent than when allowance is made for its possible presence. Despite the fact that, in all such

cases, the estimated coefficient of time in the aggregate regression turns out to be very small in absolute value, there is a definite deterioration in relative root-mean-square error, S, as compared with the same cases with that coefficient set equal to zero in the estimation of the aggregate production function. Conceivably, this is due to the fact that there are only 20 observations to begin with, so that the use of one degree of freedom to estimate a zero coefficient is not a trivial loss of efficiency. This may particularly be so when the estimation of that coefficient involves the introduction of time, a variable which makes the regressions pretty collinear, even with constant returns.

Similar considerations may also explain why the Hicks experiments tend to have higher root-mean-square errors than the corresponding capital experiments in the results below.

Given all this, our analysis of the results as to wage prediction proceeded by taking in each experiment, the wage prediction generated by the best of the aggregate production functions estimated, as indicated by the above considerations. These were the predictions generated by the estimates imposing constant returns and making the correct specification as to the presence or absence of Hicks-neutral technical change. In general, these were also the best wage predictions in terms of relative root-mean-square error. Thus the results in the next section are based on (4.1c) for the capital experiments and on (4.1d) for the Hicks experiments.

6. The Principal Results: Variance of Labor's Share and the Prediction of Wages

We now come to the principal focus of the experiments, the prediction of wages as measured by S. It is perfectly clear that there are some cases in which we would naturally expect S to be small and some in which we would expect it to be large.

Certainly, for the cases in which μ is zero or almost zero, we would expect an aggregate production function to do reasonably well. For such cases, the individual capital stocks are roughly proportional to each other and any average of them will provide a good capital index. It is true that the correct form of the aggregate production function in such cases will not be Cobb-Douglas[1], but one would expect an aggregate Cobb-Douglas to do pretty well, nevertheless. We shall refer to such cases as "low-movement" cases.

One would also expect such good performance in quite an opposite case. Suppose that one of the firms starts out very large relative to the others; suppose also that the large firm is growing relative to the others. Then the entire economy is largely dominated by that firm and output and wages are largely determined by its production function. In such a case, treating the economy as though it were just one big firm is obviously not going to be far off the mark. Note that this will be true

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[1] The correct form is the sum of the individual firms' production functions, with each $K_i(t)$ set equal to $k_i J(t)$ and the $L_i(t)$ determined by the efficiency conditions. This last makes the writing of the aggregate production function in closed form unhelpful, at best.

whether the large and growing firm has a growing capital stock or a growing technical efficiency parameter.

This sort of case is most obvious when there are only two firms and one of them is much bigger than the other; there are similar cases, however, for $n = 4$ or $n = 8$. In the two-group capital or two-group Hicks experiments (but not in the fanning experiments), the high-α firms are treated as a group, as are the low-α firms. The firms in each group start out large or small together; they also grow proportionally (a slight exception being the two-group Hicks experiments where identical growth rates are adjusted by slightly different capital exponents for different firms). Considering just one of the groups of firms, then, it forms a low-movement case and we should expect to be able to treat it more or less as a single unit. If one of those units is big and growing relative to the other, however, this closely parallels the case of two firms with one dominating the other. Again in such cases we should expect that treating the entire economy as a single firm gives fairly good results.

We shall refer to such cases as "specialization cases," observing that for $n > 2$, they have some aspects of low-movement cases as well. There are not too many such cases even for the two-group experiments. For the fanning experiments in which $n > 2$ [2], there can be no cases which

[2] For $n = 2$, fanning experiments are identical to two-group experiments.

are exactly specialization cases as just defined. On the other hand, when one group of firms starts out very big and grows relative to the other, then, even in the fanning cases, it is misleading to take the relevant rate of growth as that of the big firms relative to the small ones. The big firms dominate throughout and what matters is the rates of growth of those firms relative to each other. For $n = 4$, this is μ; for $n = 8$, this is 3μ (taking the maximum of such relative growth rates). In the results below, we have accounted for this by indexing such cases under growth rates μ and 3μ instead of 3μ and 7μ, respectively, as would be done were all n firms treated symmetrically. This means, for example, that a **fanning** case in which there are four firms with two of them big and growing, with the growth rates of the four firms being 0, .01, .02, and .03, is treated as a case of .01 relative growth rather than a case of .03 relative growth in the tables below and is thus counted with the low-movement cases.

There is also a large class of cases in which it is obvious that the wage predictions of the estimated aggregate production functions will be poor. Since those predictions are from Cobb-Douglas functions, they involve the prediction that labor's share of total output will be constant. If, in fact, labor's share has a high variance in a given experiment, one would hardly expect such wage predictions to be accurate.

While it is thus obvious that a low variance of labor's share is a necessary condition for a good set of wage predictions, it is by no means obvious that this is also a sufficient condition. Yet, by and large, we

find this to be the case. Even excluding the *a priori* obvious low-movement and specialization cases, we find that a low variance of labor's share is associated with a low relative root-mean-square error in wage prediction. This phenomenon thus occurs even in cases in which there is high relative movement and no domination by a set of proportionally growing firms, so that the underlying technical relationships do not look anything like an aggregate Cobb-Douglas (or indeed, *any* aggregate production function) in any sense. Cases which seem *a priori* similar to these generate a relatively high variance of labor's share and do badly on the relative root-mean-square error criterion.

We now present the results, postponing heuristic argument as to the plausibility of such a finding and discussion of its implications to the next section.

Each of Tables 6.1 - 6.8 gives the joint distribution of the relative root-mean-square error of the wage predictions (S) and the standard deviation of labor's share. The latter, for comparability, is divided into intervals which are .00375 in absolute value, this being .5% of .75 which is the overall mean of labor's share in all the experiments. Columns are thus labelled "Relative Standard Deviation of Labor's Share." Table 6.1A summarizes all experiments; Table 6.1C, all capital experiments; and Table 6.1H, all Hicks experiments. Tables 6.2A, C, and H, provide the same summary deleting specialization cases and all cases in which the relevant growth rates are zero or one per cent. Tables 6.3 - 6.8 (C) and 6.3 - 6.8 (H) give the results broken down by growth rates, but still excluding pure specialization cases.

In all cases, growth rates are taken as μ, so that Hicks cases are grouped with capital cases with equivalent growth rates. In the fanning cases, the relevant growth rate is taken to be that of the fastest growing firm, except in cases in which one group of firms is large and growing, in which case, as noted above, it is taken to be the growth rate of the fastest-growing large firm relative to the slowest-growing large firm.

Each box in each of the capital tables has three entries, corresponding to two-group, fanning, and all experiments, respectively; each box in each of the Hicks tables has four entries, corresponding to preliminary two-group, final two-group, fanning, and all experiments, respectively. The total of the entries for the individual experiment types is not always that given in the last line of the box because final two-group and fanning experiments are identical for $n = 2$ and are counted in each category separately but included in the total only once.

The sparseness of entries in the upper right-hand corners of the tables corresponds to the obvious fact that an aggregate Cobb-Douglas can not give a good explanation of wages when the variance of labor's share is high. The point of interest is the sparseness of entries in the lower left-hand corners, and the density of entries in the upper left-hand corners showing the cases in which the variance of labor's share is low and (for the upper corners) the relative root-mean-square error of wage prediction low. This phenomenon is particularly marked, if one observes that the left-hand axis is rather finely subdivided at the upper end, so that the first four boxes only get up to 2% relative root-mean-square error and

the first seven boxes only up to 5%, but it generally appears even within the first few boxes.[3]

Obviously, the results of the capital experiments fit the described pattern extremely tightly, there being few cases much off the principal diagonals. The results of the Hicks cases also fit it, particularly looking at the number of cases in each box and not merely at whether or not the box is empty, but not so strikingly well as the capital experiment results. This is probably due to the greater problems of multicollinearity encountered in the aggregate regressions in the Hicks experiments, as we shall discuss in the next section.

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[3] It does not seem worthwhile to compute a formal test of association. The most obvious chi-square test would involve the guaranteed emptiness of of the upper right-hand corner, a point of little interest. Moreover, the appearance of very many cases in the upper left-hand corner would be important regardless of the rest of the table. In any case, the pattern in the results is strong enough to be obvious at a glance.

TABLE 6.1A

SUMMARY OF ALL EXPERIMENTS

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
PRELIM TWO-GROUP	63	1	1				
TWO-GROUP	148	0	0				
FANNING	93	0	0				
TOTAL	254	1	1				
0.5- 1.0							
PRELIM TWO-GROUP	30	15	0				
TWO-GROUP	48	42	1				
FANNING	24	43	1				
TOTAL	93	83	1				
1.0- 1.5							
PRELIM TWO-GROUP	10	5	3	0			
TWO-GROUP	11	12	20	2			
FANNING	4	16	18	1			
TOTAL	23	30	35	2			
1.5- 2.0							
PRELIM TWO-GROUP	3	4	5	3	0		
TWO-GROUP	9	2	6	10	2		
FANNING	2	2	14	9	4		
TOTAL	13	8	22	19	4		
2.0- 3.0							
PRELIM TWO-GROUP	5	5	7	2	3	1	0
TWO-GROUP	5	4	2	6	8	5	1
FANNING	5	2	3	12	7	8	4
TOTAL	15	11	12	15	13	12	4
3.0- 4.0							
PRELIM TWO-GROUP	0	3	0	3	1	1	1
TWO-GROUP	4	4	2	1	3	0	3
FANNING	4	5	2	3	1	1	13
TOTAL	6	12	4	7	4	2	16
4.0- 5.0							
PRELIM TWO-GROUP	0	2	1	1	1	0	1
TWO-GROUP	1	0	3	1	0	0	3
FANNING	4	2	2	1	1	1	9
TOTAL	5	4	5	3	2	1	11
5.0-10.0							
PRELIM TWO-GROUP	3	1	1	2	2	1	1
TWO-GROUP	0	5	2	5	2	3	3
FANNING	1	7	3	1	4	7	16
TOTAL	4	10	5	7	6	9	18
10.0-20.0							
PRELIM TWO-GROUP		0	0	1	0	1	4
TWO-GROUP		1	0	0	0	2	2
FANNING		1	4	2	1	0	5
TOTAL		2	4	3	1	3	11
ABOVE 20							
PRELIM TWO-GROUP	0	0	0	0	0	0	0
TWO-GROUP	0	1	0	0	0	0	1
FANNING	1	0	4	2	1	2	11
TOTAL	1	1	4	2	1	2	11

ITEMS SOMETIMES DO NOT ADD TO TOTALS BECAUSE EACH TWO-FIRM EXPERIMENT WAS COUNTED IN BOTH CATEGORIES, BUT ADDED IN TO TOTAL ONLY ONCE.

TABLE 6.1C

SUMMARY OF ALL CAPITAL EXPERIMENTS

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
TWO-GROUP	87						
FANNING	52						
TOTAL	108						
0.5- 1.0							
TWO-GROUP	27	30	1				
FANNING	8	28	1				
TOTAL	34	47	1				
1.0- 1.5							
TWO-GROUP		4	17	2			
FANNING		6	17	1			
TOTAL		10	29	2			
1.5- 2.0							
TWO-GROUP		0	0	10	2		
FANNING		1	8	6	3		
TOTAL		1	8	13	3		
2.0- 3.0							
TWO-GROUP		0	0	1	5	5	1
FANNING		1	3	5	5	7	4
TOTAL		1	3	5	7	10	4
3.0- 4.0							
TWO-GROUP			0	0	1		3
FANNING			1	2	0		11
TOTAL			1	2	1		13
4.0- 5.0							
TWO-GROUP				0	0		1
FANNING				1	1		8
TOTAL				1	1		8
5.0-10.0							
TWO-GROUP						0	1
FANNING						2	14
TOTAL						2	14
10.0-20.0							
TWO-GROUP							0
FANNING							1
TOTAL							1
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

ITEMS SOMETIMES DO NOT ADD TO TOTALS BECAUSE EACH TWO-FIRM EXPERIMENT WAS COUNTED IN BOTH CATEGORIES, BUT ADDED IN TO TOTAL ONLY ONCE.

TABLE 6.1H

SUMMARY OF ALL HICKS EXPERIMENTS

REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT) \ RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
PRELIM TWO-GROUP	63	1	1				
TWO-GROUP	61	0	0				
FANNING	41	0	0				
TOTAL	146	1	1				
0.5- 1.0							
PRELIM TWO-GROUP	30	15					
TWO-GROUP	21	12					
FANNING	16	15					
TOTAL	59	36					
1.0- 1.5							
PRELIM TWO-GROUP	10	5	3				
TWO-GROUP	11	8	3				
FANNING	4	10	1				
TOTAL	23	20	6				
1.5- 2.0							
PRELIM TWO-GROUP	3	4	5	3	0		
TWO-GROUP	9	2	6	0	0		
FANNING	2	1	6	3	1		
TOTAL	13	7	14	6	1		
2.0- 3.0							
PRELIM TWO-GROUP	5	5	7	2	3	1	
TWO-GROUP	5	4	2	5	3	0	
FANNING	5	1	0	7	2	1	
TOTAL	15	10	9	10	6	2	
3.0- 4.0							
PRELIM TWO-GROUP	0	3	0	3	1	1	1
TWO-GROUP	4	4	2	1	2	0	0
FANNING	4	5	1	1	1	1	2
TOTAL	6	12	3	5	3	2	3
4.0- 5.0							
PRELIM TWO-GROUP	0	2	1	1	1	0	1
TWO-GROUP	1	0	3	1	0	0	2
FANNING	4	2	2	0	0	1	1
TOTAL	5	4	5	2	1	1	3
5.0-10.0							
PRELIM TWO-GROUP	3	1	1	2	2	1	1
TWO-GROUP	0	5	2	5	2	3	2
FANNING	1	7	3	1	4	5	2
TOTAL	4	10	5	7	6	7	4
10.0-20.0							
PRELIM TWO-GROUP		0	0	1	0	1	4
TWO-GROUP		1	0	0	0	2	2
FANNING		1	4	2	1	0	5
TOTAL		2	4	3	1	3	10
ABOVE 20							
PRELIM TWO-GROUP	0	0	0	0	0	0	0
TWO-GROUP	0	1	0	0	0	0	1
FANNING	1	0	4	2	1	2	11
TOTAL	1	1	4	2	1	2	11

ITEMS SOMETIMES DO NOT ADD TO TOTALS BECAUSE EACH TWO-FIRM EXPERIMENT WAS COUNTED IN BOTH CATEGORIES, BUT ADDED IN TO TOTAL ONLY ONCE.

TABLE 6.2A

SUMMARY OF ALL NON-TRIVIAL EXPERIMENTS

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
TWO-GROUP	58	1	1				
FANNING	39	0	0				
TOTAL	84	1	1				
0.5- 1.0							
TWO-GROUP	45	36					
FANNING	15	33					
TOTAL	55	58					
1.0- 1.5							
TWO-GROUP	15	14	12				
FANNING	3	15	13				
TOTAL	17	26	22				
1.5- 2.0							
TWO-GROUP	10	6	4	3	0		
FANNING	2	2	12	6	2		
TOTAL	11	8	14	8	2		
2.0- 3.0							
TWO-GROUP	7	9	6	4	7	6	1
FANNING	5	2	3	8	4	8	4
TOTAL	12	11	9	10	9	12	4
3.0- 4.0							
TWO-GROUP	1	7	0	2	4	1	4
FANNING	2	5	2	3	1	1	13
TOTAL	3	12	2	5	4	2	16
4.0- 5.0							
TWO-GROUP	1	2	1	2	1	0	4
FANNING	4	2	1	1	1	1	9
TOTAL	5	4	2	3	2	1	11
5.0-10.0							
TWO-GROUP	3	6	1	4	2	4	4
FANNING	1	7	3	1	3	7	16
TOTAL	4	10	3	4	4	9	18
10.0-20.0							
TWO-GROUP		1	0	0	0	3	6
FANNING		1	4	2	1	0	6
TOTAL		2	4	2	1	3	11
ABOVE 20							
TWO-GROUP	0	1	0	0	0	0	1
FANNING	1	0	4	2	1	2	11
TOTAL	1	1	4	2	1	2	11

DOES NOT INCLUDE GROWTH RATES OF -1 TO +1 PERCENT OR PURE (TWO-GROUP) SPECIALIZATION CASES.
ITEMS SOMETIMES DO NOT ADD TO TOTALS BECAUSE EACH TWO-FIRM EXPERIMENT WAS COUNTED IN BOTH CATEGORIES, BUT ADDED IN TO TOTAL ONLY ONCE.

TABLE 6.2C

SUMMARY OF NON-TRIVIAL CAPITAL EXPERIMENTS

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
TWO-GROUP	24						
FANNING	21						
TOTAL	36						
0.5- 1.0							
TWO-GROUP	17	22					
FANNING	6	23					
TOTAL	22	37					
1.0- 1.5							
TWO-GROUP		3	9				
FANNING		6	13				
TOTAL		9	19				
1.5- 2.0							
TWO-GROUP		0	[illegible]	3	0		
FANNING		1	9	3	1		
TOTAL		1	8	5	1		
2.0- 3.0							
TWO-GROUP		0	0	1	4	5	1
FANNING		1	3	5	4	7	4
TOTAL		1	3	5	6	10	4
3.0- 4.0							
TWO-GROUP			0	0	1		3
FANNING			1	2	0		11
TOTAL			1	2	1		13
4.0- 5.0							
TWO-GROUP				0	0		1
FANNING				1	1		8
TOTAL				1	1		8
5.0-10.0							
TWO-GROUP						0	1
FANNING						2	14
TOTAL						2	14
10.0-20.0							
TWO-GROUP							0
FANNING							1
TOTAL							1
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE GROWTH RATES OF -1 TO +1 PERCENT OR PURE (TWO-GROUP) SPECIALIZATION CASES.
ITEMS SOMETIMES DO NOT ADD TO TOTALS BECAUSE EACH TWO-FIRM EXPERIMENT WAS COUNTED IN BOTH CATEGORIES, BUT ADDED IN TO TOTAL ONLY ONCE.

TABLE 6.2H

SUMMARY OF NON-TRIVIAL HICKS EXPERIMENTS

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
TWO-GROUP	34	1	1				
FANNING	18	0	0				
TOTAL	49	1	1				
0.5- 1.0							
TWO-GROUP	28	14					
FANNING	9	10					
TOTAL	33	21					
1.0- 1.5							
TWO-GROUP	15	11	3				
FANNING	3	9	0				
TOTAL	17	17	3				
1.5- 2.0							
TWO-GROUP	10	6	4	0	0		
FANNING	2	1	4	3	1		
TOTAL	11	7	6	3	1		
2.0- 3.0							
TWO-GROUP	7	9	6	3	3	1	
FANNING	5	1	0	3	0	1	
TOTAL	12	10	6	5	3	2	
3.0- 4.0							
TWO-GROUP	1	7	0	2	3	1	1
FANNING	2	5	1	1	1	1	2
TOTAL	3	12	1	3	3	2	3
4.0- 5.0							
TWO-GROUP	1	2	1	2	1	0	3
FANNING	4	2	1	0	0	1	1
TOTAL	5	4	2	2	1	1	3
5.0-10.0							
TWO-GROUP	3	6	1	4	2	4	3
FANNING	1	7	3	1	3	5	2
TOTAL	4	10	3	4	4	7	4
10.0-20.0							
TWO-GROUP		1	0	0	0	3	6
FANNING		1	4	2	1	0	5
TOTAL		2	4	2	1	3	10
ABOVE 20							
TWO-GROUP	0	1	0	0	0	0	1
FANNING	1	0	4	2	1	2	11
TOTAL	1	1	4	2	1	2	11

DOES NOT INCLUDE GROWTH RATES OF -1 TO +1 PERCENT OR PURE (TWO-GROUP) SPECIALIZATION CASES.
ITEMS SOMETIMES DO NOT ADD TO TOTALS BECAUSE EACH TWO-FIRM EXPERIMENT WAS COUNTED IN BOTH CATEGORIES, BUT ADDED IN TO TOTAL ONLY ONCE.

TABLE 6.3C

CAPITAL EXPERIMENTS, 0- 1 PERCENT GROWTH

RELATIVE STANO DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREOICTING WAGES (PERCENT)							
0.0- 0.5							
TWO-GROUP	22						
FANNING	16						
TOTAL	31						
0.5- 1.0							
TWO-GROUP	5	7	1				
FANNING	2	5	1				
TOTAL	7	9	1				
1.0- 1.5							
TWO-GROUP		1	2	1			
FANNING		0	3	1			
TOTAL		1	4	1			
1.5- 2.0							
TWO-GROUP				2	1		
FANNING				1	1		
TOTAL				3	1		
2.0- 3.0							
TWO-GROUP							
FANNING							
TOTAL							
3.0- 4.0							
TWO-GROUP							
FANNING							
TOTAL							
4.0- 5.0							
TWO-GROUP							
FANNING							
TOTAL							
5.0-10.0							
TWO-GROUP							
FANNING							
TOTAL							
10.0-20.0							
TWO-GROUP							
FANNING							
TOTAL							
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

OOES NOT INCLUOE PURE (TWC-GROUP) SPECIALIZATION CASES.

TABLE 6.4C

CAPITAL EXPERIMENTS, 2- 3 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE FRROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
TWO-GROUP	21						
FANNING	17						
TOTAL	30						
0.5- 1.0							
TWO-GROUP	9	9					
FANNING	2	8					
TOTAL	10	14					
1.0- 1.5							
TWO-GROUP		1					
FANNING		1					
TOTAL		2					
1.5- 2.0							
TWO-GROUP					0		
FANNING					1		
TOTAL					1		
2.0- 3.0							
TWO-GROUP				1	3	4	
FANNING				1	2	4	
TOTAL				1	4	6	
3.0- 4.0							
TWO-GROUP							
FANNING							
TOTAL							
4.0- 5.0							
TWO-GROUP							
FANNING							
TOTAL							
5.0-10.0							
TWO-GROUP							
FANNING							
TOTAL							
10.0-20.0							
TWO-GROUP							
FANNING							
TOTAL							
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.5C

CAPITAL EXPERIMENTS, 4- 5 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
TWO-GROUP	3						
FANNING	2						
TOTAL	4						
0.5- 1.0							
TWO-GROUP	8	13					
FANNING	1	6					
TOTAL	9	14					
1.0- 1.5							
TWO-GROUP		2	9				
FANNING		1	5				
TOTAL		3	11				
1.5- 2.0							
TWO-GROUP				3			
FANNING				1			
TOTAL				3			
2.0- 3.0							
TWO-GROUP					1	1	1
FANNING					1	0	2
TOTAL					1	1	2
3.0- 4.0							
TWO-GROUP					1		3
FANNING					0		2
TOTAL					1		4
4.0- 5.0							
TWO-GROUP							1
FANNING							1
TOTAL							1
5.0-10.0							
TWO-GROUP							1
FANNING							1
TOTAL							1
10.0-20.0							
TWO-GROUP							
FANNING							
TOTAL							
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.6C

CAPITAL EXPERIMENTS, 6-10 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
TWO-GROUP	0						
FANNING	2						
TOTAL	2						
0.5- 1.0							
TWO-GROUP	0	0					
FANNING	2	8					
TOTAL	2	8					
1.0- 1.5							
TWO-GROUP		0	0				
FANNING		4	4				
TOTAL		4	4				
1.5- 2.0							
TWO-GROUP			0	0			
FANNING			1	1			
TOTAL			1	1			
2.0- 3.0							
TWO-GROUP				0		0	
FANNING				2		2	
TOTAL				2		2	
3.0- 4.0							
TWO-GROUP							0
FANNING							5
TOTAL							5
4.0- 5.0							
TWO-GROUP							0
FANNING							1
TOTAL							1
5.0-10.0							
TWO-GROUP							
FANNING							
TOTAL							
10.0-20.0							
TWO-GROUP							
FANNING							
TOTAL							
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.7C

CAPITAL EXPERIMENTS.11-15 PERCENT GROWTH

REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT) \ RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
TWO-GROUP							
FANNING							
TOTAL							
0.5- 1.0							
TWO-GROUP	0	0					
FANNING	1	1					
TOTAL	1	1					
1.0- 1.5							
TWO-GROUP			0				
FANNING			4				
TOTAL			4				
1.5- 2.0							
TWO-GROUP		0	0	0			
FANNING		1	6	1			
TOTAL		1	6	1			
2.0- 3.0							
TWO-GROUP		0	0	0		0	
FANNING		1	1	2		1	
TOTAL		1	1	2		1	
3.0- 4.0							
TWO-GROUP							0
FANNING							3
TOTAL							3
4.0- 5.0							
TWO-GROUP							0
FANNING							5
TOTAL							5
5.0-10.0							
TWO-GROUP							0
FANNING							5
TOTAL							5
10.0-20.0							
TWO-GROUP							
FANNING							
TOTAL							
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.8C

CAPITAL EXPERIMENTS, 16-35 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
TWO-GROUP							
FANNING							
TOTAL							
0.5- 1.0							
TWO-GROUP							
FANNING							
TOTAL							
1.0- 1.5							
TWO-GROUP							
FANNING							
TOTAL							
1.5- 2.0							
TWO-GROUP			0				
FANNING			1				
TOTAL			1				
2.0- 3.0							
TWO-GROUP			0		0		0
FANNING			2		1		2
TOTAL			2		1		2
3.0- 4.0							
TWO-GROUP			0	0			0
FANNING			1	2			1
TOTAL			1	2			1
4.0- 5.0							
TWO-GROUP				0	0		0
FANNING				1	1		1
TOTAL				1	1		1
5.0-10.0							
TWO-GROUP						0	0
FANNING						2	8
TOTAL						2	8
10.0-20.0							
TWO-GROUP							0
FANNING							1
TOTAL							1
ABOVE 20							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.3H

HICKS EXPERIMENTS, 0- 1 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
PRELIM TWO-GROUP	9						
TWO-GROUP	20						
FANNING	14						
TOTAL	37						
0.5- 1.0							
PRELIM TWO-GROUP	2	3					
TWO-GROUP	6	8					
FANNING	4	5					
TOTAL	11	13					
1.0- 1.5							
PRELIM TWO-GROUP		1	1				
TWO-GROUP		1	1				
FANNING		1	1				
TOTAL		3	2				
1.5- 2.0							
PRELIM TWO-GROUP			2				
TWO-GROUP			2				
FANNING			2				
TOTAL			5				
2.0- 3.0							
PRELIM TWO-GROUP				0	0		
TWO-GROUP				2	1		
FANNING				2	1		
TOTAL				3	1		
3.0- 4.0							
PRELIM TWO-GROUP							
TWO-GROUP							
FANNING							
TOTAL							
4.0- 5.0							
PRELIM TWO-GROUP							
TWO-GROUP							
FANNING							
TOTAL							
5.0-10.0							
PRELIM TWO-GROUP				0			
TWO-GROUP				1			
FANNING				0			
TOTAL				1			
10.0-20.0							
PRELIM TWO-GROUP							
TWO-GROUP							
FANNING							
TOTAL							
ABOVE 20							
PRELIM TWO-GROUP							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.4B

HICKS EXPERIMENTS, 2- 3 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
PRELIM TWO-GROUP	0						
TWO-GROUP	11						
FANNING	11						
TOTAL	18						
0.5- 1.0							
PRELIM TWO-GROUP	0	0					
TWO-GROUP	6	2					
FANNING	4	5					
TOTAL	7	5					
1.0- 1.5							
PRELIM TWO-GROUP	0	0					
TWO-GROUP	5	3					
FANNING	1	2					
TOTAL	5	4					
1.5- 2.0							
PRELIM TWO-GROUP	0	0					
TWO-GROUP	7	1					
FANNING	1	0					
TOTAL	7	1					
2.0- 3.0							
PRELIM TWO-GROUP	0	0			0		
TWO-GROUP	1	1			1		
FANNING	2	0			0		
TOTAL	3	1			1		
3.0- 4.0							
PRELIM TWO-GROUP		0		0	0		
TWO-GROUP		3		0	2		
FANNING		0		1	1		
TOTAL		3		1	2		
4.0- 5.0							
PRELIM TWO-GROUP	0						
TWO-GROUP	0						
FANNING	1						
TOTAL	1						
5.0-10.0							
PRELIM TWO-GROUP		0		0	0	0	
TWO-GROUP		0		1	0	3	
FANNING		1		1	1	3	
TOTAL		1		1	1	4	
10.0-20.0							
PRELIM TWO-GROUP				0		0	
TWO-GROUP				0		1	
FANNING				1		0	
TOTAL				1		1	
ABOVE 20							
PRELIM TWO-GROUP							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.5H

HICKS EXPERIMENTS, 4- 5 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
PRELIM TWO-GROUP	10						
TWO-GROUP	0						
FANNING	2						
TOTAL	12						
0.5- 1.0							
PRELIM TWO-GROUP	5	3					
TWO-GROUP	3	1					
FANNING	1	1					
TOTAL	8	4					
1.0- 1.5							
PRELIM TWO-GROUP	1	1	0				
TWO-GROUP	3	4	1				
FANNING	1	2	0				
TOTAL	5	5	1				
1.5- 2.0							
PRELIM TWO-GROUP		0	0	0			
TWO-GROUP		1	3	0			
FANNING		0	2	1			
TOTAL		1	3	1			
2.0- 3.0							
PRELIM TWO-GROUP	0	0	1	1			
TWO-GROUP	3	3	2	1			
FANNING	1	0	0	2			
TOTAL	4	3	3	3			
3.0- 4.0							
PRELIM TWO-GROUP	0	0	0	1			
TWO-GROUP	1	1	0	0			
FANNING	0	0	1	0			
TOTAL	1	1	1	1			
4.0- 5.0							
PRELIM TWO-GROUP	0		0	0			0
TWO-GROUP	1		1	1			2
FANNING	0		0	0			1
TOTAL	1		1	1			2
5.0-10.0							
PRELIM TWO-GROUP		0	0	1	0		0
TWO-GROUP		5	1	1	1		2
FANNING		3	1	0	1		1
TOTAL		5	1	2	1		2
10.0-20.0							
PRELIM TWO-GROUP		0	0			0	0
TWO-GROUP		1	0			1	2
FANNING		0	1			0	1
TOTAL		1	1			1	2
ABOVE 20							
PRELIM TWO-GROUP		0					0
TWO-GROUP		1					1
FANNING		0					1
TOTAL		1					1

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.6H

HICKS EXPERIMENTS, 6-10 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)							
0.0- 0.5							
PRELIM TWO-GROUP	10						
TWO-GROUP	0						
FANNING	1						
TOTAL	11						
0.5- 1.0							
PRELIM TWO-GROUP	5	1					
TWO-GROUP	0	0					
FANNING	2	3					
TOTAL	7	4					
1.0- 1.5							
PRELIM TWO-GROUP	1	1					
TWO-GROUP	0	0					
FANNING	1	3					
TOTAL	2	4					
1.5- 2.0							
PRELIM TWO-GROUP	1	0	0	0			
TWO-GROUP	0	0	0	0			
FANNING	0	1	2	1			
TOTAL	1	1	2	1			
2.0- 3.0							
PRELIM TWO-GROUP	0	1		0	1		
TWO-GROUP	0	0		0	0		
FANNING	1	0		1	0		
TOTAL	1	1		1	1		
3.0- 4.0							
PRELIM TWO-GROUP	0	0			1	0	
TWO-GROUP	0	0			0	0	
FANNING	1	3			0	1	
TOTAL	1	3			1	1	
4.0- 5.0							
PRELIM TWO-GROUP	0			1		0	
TWO-GROUP	0			0		0	
FANNING	2			0		1	
TOTAL	2			1		1	
5.0-10.0							
PRELIM TWO-GROUP	0	0	0		1	0	0
TWO-GROUP	0	0	0		0	0	0
FANNING	1	1	1		1	1	1
TOTAL	1	1	1		2	1	1
10.0-20.0							
PRELIM TWO-GROUP					0		0
TWO-GROUP					0		0
FANNING					1		1
TOTAL					1		1
ABOVE 20							
PRELIM TWO-GROUP							
TWO-GROUP							
FANNING							
TOTAL							

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.7H

HICKS EXPERIMENTS, 11-15 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
PRELIM TWO-GROUP	3	1					
TWO-GROUP	0	0					
FANNING	2	0					
TOTAL	5	1					
0.5- 1.0							
PRELIM TWO-GROUP	6	2					
TWO-GROUP	0	0					
FANNING	1	0					
TOTAL	7	2					
1.0- 1.5							
PRELIM TWO-GROUP	2	0					
TWO-GROUP	0	0					
FANNING	0	1					
TOTAL	2	1					
1.5- 2.0							
PRELIM TWO-GROUP	1	1		0	0		
TWO-GROUP	0	0		0	0		
FANNING	1	0		1	1		
TOTAL	2	1		1	1		
2.0- 3.0							
PRELIM TWO-GROUP	1	1			1	1	
TWO-GROUP	0	0			0	0	
FANNING	1	1			0	1	
TOTAL	2	2			1	2	
3.0- 4.0							
PRELIM TWO-GROUP	0	1				1	
TWO-GROUP	0	0				0	
FANNING	1	2				0	
TOTAL	1	3				1	
4.0- 5.0							
PRELIM TWO-GROUP		1	0				
TWO-GROUP		0	0				
FANNING		1	1				
TOTAL		2	1				
5.0-10.0							
PRELIM TWO-GROUP						1	
TWO-GROUP						0	
FANNING						1	
TOTAL						2	
10.0-20.0							
PRELIM TWO-GROUP		0	0	0			0
TWO-GROUP		0	0	0			0
FANNING		1	2	1			2
TOTAL		1	2	1			2
ABOVE 20							
PRELIM TWO-GROUP	0		0	0		0	0
TWO-GROUP	0		0	0		0	0
FANNING	1		3	1		1	4
TOTAL	1		3	1		1	4

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

TABLE 6.8B

HICKS EXPERIMENTS,16-35 PERCENT GROWTH

RELATIVE STAND DEV OF LABOR'S SHARE (PERCENT) / REL ROOT MEAN SQUARE ERROR IN PREDICTING WAGES (PERCENT)	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	ABOVE 3.0
0.0- 0.5							
PRELIM TWO-GROUP	0		1				
TWO-GROUP	0		0				
FANNING	2		0				
TOTAL	2		1				
0.5- 1.0							
PRELIM TWO-GROUP	3	5					
TWO-GROUP	0	0					
FANNING	1	1					
TOTAL	4	6					
1.0- 1.5							
PRELIM TWO-GROUP	3	2	2				
TWO-GROUP	0	0	0				
FANNING	0	1	0				
TOTAL	3	3	2				
1.5- 2.0							
PRELIM TWO-GROUP	1	3	1				
TWO-GROUP	0	0	0				
FANNING	0	0	0				
TOTAL	1	3	1				
2.0- 3.0							
PRELIM TWO-GROUP	2	3	3	1			
TWO-GROUP	0	0	0	0			
FANNING	0	0	0	0			
TOTAL	2	3	3	1			
3.0- 4.0							
PRELIM TWO-GROUP		2		1			1
TWO-GROUP		0		0			0
FANNING		0		0			2
TOTAL		2		1			3
4.0- 5.0							
PRELIM TWO-GROUP	0	1			1		1
TWO-GROUP	0	0			0		0
FANNING	1	1			0		0
TOTAL	1	2			1		1
5.0-10.0							
PRELIM TWO-GROUP	3	1	0	1			1
TWO-GROUP	0	0	0	0			0
FANNING	0	2	1	0			0
TOTAL	3	3	1	1			1
10.0-20.0							
PRELIM TWO-GROUP			0			1	4
TWO-GROUP			0			0	0
FANNING			1			0	1
TOTAL			1			1	5
ABOVE 20							
PRELIM TWO-GROUP			0	0	0	0	0
TWO-GROUP			0	0	0	0	0
FANNING			1	1	1	1	6
TOTAL			1	1	1	1	6

DOES NOT INCLUDE PURE (TWO-GROUP) SPECIALIZATION CASES.

7. Discussion of the Results

Why should an aggregate Cobb-Douglas do well in explaining wages when labor's share has a small variance? With the benefit of hindsight it is possible to give a fairly convincing heuristic explanation as to the reasonableness of our principal result for these experiments, showing that it is, in part, a consequence of the subsidiary results discussed earlier. Indeed, that heuristic explanation strikes me as so convincing that I hesitate to give it lest the reader mistake it for a proof and believe that all the experiments were unnecessary. It is not a proof, however, and the reader should bear in mind the fact that in experiments such as this in which the data are all internally generated with little stochastic element, any organizing principle for the results which can be made to seem very reasonable will also misleadingly appear afterwards to have been deducible without the experiments.

Consider the polar case in which labor's share just happens to be constant. Call that share $\bar{\alpha}$. Then wages and output per man are certainly related by:

$$w(t) = (1/\bar{\alpha})(Y^*(t)/L(t)) \quad . \tag{7.1}$$

We have already noted, however, that our regressions invariably give an extremely close fit to log $(Y^*(t)/L(t))$, so that we would expect to do well in explaining wages in such a case with labor's share constant, provided that we obtained a decent estimate of $\bar{\alpha}$.

It is clear, however, that since the marginal product of labor is

equal to w(t) in every firm:

(7.2)
$$\bar{\alpha} = \frac{w(t)L(t)}{Y^*(t)} = \frac{\sum_{i=1}^{n} w(t)L_i(t)}{\sum_{i=1}^{n} Y_i^*(t)}$$

$$= \frac{\sum_{i=1}^{n} \alpha_i \left(\frac{Y_i^*(t)}{L_i(t)}\right) L_i(t)}{\sum_{i=1}^{n} Y_i^*(t)} = \frac{\sum_{i=1}^{n} \alpha_i Y_i^*(t)}{\sum_{i=1}^{n} Y_i^*(t)} ,$$

where $Y_i^*(t)$ is the output produced by the ith firm when labor has been efficiently allocated. Thus $\bar{\alpha}$ is a weighted average of the individual α_i, the weights being proportional to the individual firm outputs.[1] We have already seen, however, that the estimate of labor's share given by the aggregate Cobb-Douglas α, like the other parameters is usually within the range of the corresponding micro-parameters, in this case the α_i, so that it is plausible that when labor's share is constant, the estimate of α will come close to $\bar{\alpha}$ (although the α is not a weighted average of the α_i and does not have the same weights).

---

[1] This shows that labor's share will be roughly constant if relative outputs are roughly constant, which will occur, for example, if the 3% labor trend and the coefficients just happen to offset the other trends because increasing amounts of labor are assigned to relatively slow growing firms. If n = 2, such rough constancy of relative outputs is the only way in which labor's share can be roughly constant; for n > 2, there are other ways.

It thus follows from the success of the aggregate regressions in explaining output per man and the plausible appearance of α as within the range of the α_i that wages will tend to be well predicted when labor's share is roughly constant. Indeed, the exceptions to this general rule observed in the tables reporting the Hicks experiments are typically cases in which the estimated α fails to lie in the indicated range, an occurence which is probably due to the multicollinearity which affects the Hicks regressions considerably more than the capital regressions.[2]

This argument makes it very plausible that in these experiments rough constancy of labor's share should lead to a situation in which an aggregate Cobb-Douglas gives generally good results including good wage predictions, even though the underlying technical relationships are not consistent with the existence of any aggregate production function and even though there is considerable relative movement in the underlying firm variables. Whether such an argument or such results have much bearing on a real world in which underlying relationships are more complicated and aggregation takes place over labor and output as well as capital is necessarily a somewhat open question. The suggestion is clear, however, that labor's share is not roughly constant because the diverse technical relationships of modern economies are truly representable by an aggregate Cobb-Douglas but rather that such relationships appear to be representable by an aggregate Cobb-Douglas <u>because</u> labor's share happens to be roughly constant. If this is so, then the reason for such rough constancy becomes

---

[2] Recall that the results presented in the tables are those from regressions including t for the Hicks experiments and excluding t for the capital experiments.

an important subject for further research.[3]

If one rejects the Cobb-Douglas form in favor of an alternative aggregate production function, the suggestion (though less direct this time) remains that the apparent success of such a function in explaining wages occurs not because such functions really represent the true state of technology but rather because their implications as to the stylized facts of wage behavior agree with what happens to be going on anyway. The development of the CES, for example, began with the observation that wages are an increasing function of output per man and that the function involved can be approximated by one linear in the logarithms.[4] The present results suggest (but only suggest) that the explanation of that wage-output per man relationship may not be in the existence of an aggregate CES but rather that the apparent existence of an aggregate CES may be explained by that relationship.

---

[3] It is hard to believe that it is explained along the lines of (7.2) by the existence of underlying Cobb-Douglas functions at the micro level together with rough constancy of relative outputs. Relative outputs do not seem very constant and, if they were, one would still want to know why.

[4] See Arrow, Chenery, Minhas, and Solow [1].